

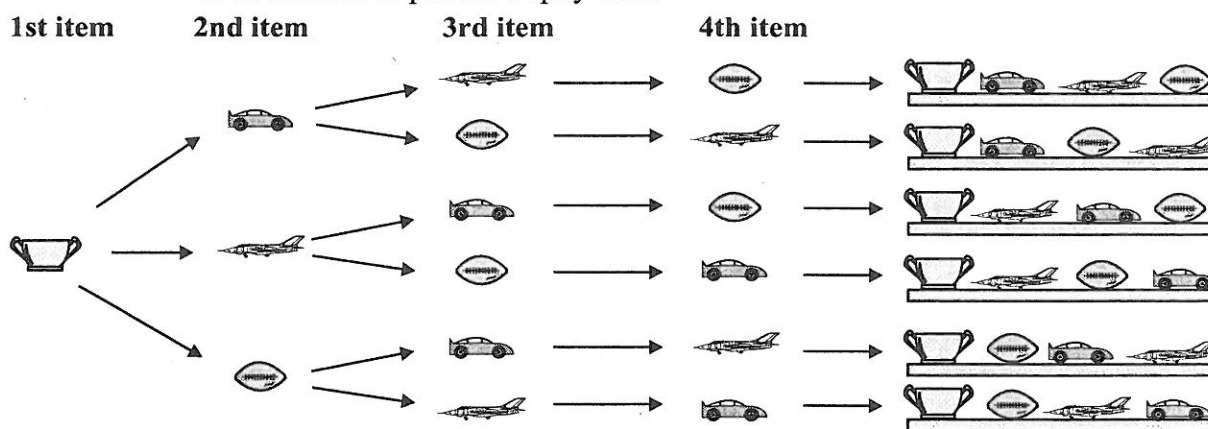
# Chapter 19: Permutations and Combinations

## PERMUTATIONS

A **permutation** is an arrangement of items in a specific order. The formula  ${}_n P_r = \frac{n!}{(n-r)!}$  is the formula for permutations.  $n$  is the number you have to choose from, and  $r$  is the number of objects you want to arrange. If a problem asks how many ways you can arrange 6 books on a bookshelf, it is asking you how many permutations there are for 6 items.

**EXAMPLE 1:** Ron has 4 items: a model airplane, a trophy, an autographed football, and a toy sports car. How many ways can he arrange the 4 items on a shelf?

**Solution:** The diagram below shows the permutations for arranging the 4 items on a shelf if he chooses to put the trophy first.



There are 6 permutations. Next, you could construct a tree diagram of permutations choosing the model car first. That tree diagram would also have 6 permutations. Then, you could construct a tree diagram choosing the airplane first. Finally, you could construct a pyramid choosing the football first. You would then have a total of 4 tree diagrams, each having 6 permutations. The total number of permutations is  $6 \times 4 = 24$ . There are 24 ways to arrange the 4 items on a bookshelf.

You probably don't want to draw tree diagrams for every permutation problem. For the problem above, Ron has 4 items to arrange. Therefore, multiply  $4 \times 3 \times 2 \times 1 = 24$ . Another way of expressing this calculation is  $4!$ , stated as 4 factorial.  $4! = 4 \times 3 \times 2 \times 1$ .

**Note:** To find the permutation using the formula,  ${}_n P_r = \frac{n!}{(n-r)!}$ .  $n$  is the number you have to choose from, 4, and  $r$  is the number of objects you want to arrange, 4.

$${}_4 P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4! = 4 \times 3 \times 2 \times 1 = 24.$$

(Remember that  $0! = 1$ .)

**EXAMPLE 2:** How many ways can you line up 6 students?

**Solution:** The number of permutations for 6 students =  $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ . There are 6 choices for the first position, 5 for the second, 4 for the third, 3 for the fourth, 2 for the fifth, and 1 for the sixth.

**Note:** The number of permutations for 6 students =  ${}_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6!}{1} = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ .

**EXAMPLE 3:** Shelley and her mom, dad, and brother are having cake for her birthday. Since it is Shelley's birthday, she gets a piece first. How many ways are there to pass out the pieces of cake?

**Solution:** Since Shelley gets the first piece, the first spot is fixed. The second, third, and fourth spots are not fixed, and anyone left can be in one of the three spots.

Spot	1	2	3	4
Choices of people	1	3	2	1

Now, multiply the choices together,  $1 \times 3 \times 2 \times 1 = 6$  ways to pass out cake.

**Note:** You can also use the permutation formula, but fix the first spot and just arrange the last three spots.  $1 \times {}_3P_3 = 1 \times \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$  ways

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| 1. How many ways can you arrange five books on a bookshelf?<br>_____  | 5. At Sam's party, the DJ has four song requests. In how many different orders can he play the four songs?<br>_____                       |
| 2. Myra has six novels to arrange on a bookshelf. How many ways can she arrange the novels?<br>_____  | 6. Yvette has five comic books. How many different ways can she stack the comic books?<br>_____   |
| 3. Seven sprinters signed up for the 100-meter dash. How many ways can the seven sprinters line up on the start line?<br>_____  | 7. Sandra's couch can hold three people. How many ways can she and her two friends sit on the couch?<br>_____                             |
| 4. Keri wants an ice cream cone with one scoop of chocolate, one scoop of vanilla, and one scoop of strawberry. How many ways can the scoops be arranged on the cone if the top flavor is chocolate?<br>_____ | 8. How many ways can you arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and always have 3 at position 1 and 10 at position 5?<br>_____ |

## MORE PERMUTATIONS

The formula  ${}_nP_r = \frac{n!}{(n-r)!}$  can also be used if you are trying to arrange a specific number of objects, but have more than you want to arrange.  $n$  is the number you have to choose from, and  $r$  is the number of objects you want to arrange.

**EXAMPLE:** If there are 6 students, how many ways can you line up any 4 of them?

**Step 1:** Find all your variables. The formula is  ${}_nP_r = \frac{n!}{(n-r)!}$ .  
 $n$  (the number you have to choose from) = 6  
 $r$  (the number of objects you want to arrange) = 4

**Step 2:** Plug into the formula.  
 ${}_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{2 \times 1} = 6 \times 5 \times 4 \times 3 = 360$ .

There are 360 ways to line up 4 of the 6 students.

**Find the number of permutations for each of the problems below.**

- How many ways can you arrange four out of eight books on a shelf?
- How many 3 digit numbers can be made using the numbers 2, 3, 5, 8, and 9?
- How many ways can you line up four students out of a class of twenty?
- Kim worked in the linen department of a store. Eight new colors of towels came in. Her job was to line up the new towels on a long shelf. How many ways could she arrange the eight colors?
- Terry's CD player holds 5 CDs. Terry owns 12 CDs. How many different ways can he arrange his CDs in the CD player?
- Erik has eleven shirts he wears to school. How many ways can he choose a different shirt to wear on Monday, Tuesday, Wednesday, Thursday, and Friday?
- Deb has a box of twelve markers. The art teacher told her to choose three markers and line them up on her desk. How many ways can she line up three markers from the twelve?
- Jeff went into an ice cream store serving 32 flavors of ice cream. He wanted a cone with two different flavors. How many ways could he order two scoops of ice cream, one on top of the other?
- In how many ways can you arrange any three letters from the 26 letters in the alphabet?

## COMBINATIONS

In a **combination**, the order does not matter. In a **permutation**, if someone picked two letters of the alphabet, **k, m** and **m, k**, they would be considered 2 different permutations. In a **combination**, **k, m** and **m, k** would be the same combination. A different order does not make a new combination. The formula for combinations is  ${}_n C_r = \frac{n!}{(n-r)! r!}$  where  $n$  is the total number of objects you choose from and  $r$  is the number that you choose to arrange.

**EXAMPLE:** How many combinations of three letters from the set {a, b, c, d, e} are there?

**Step 1:** Find the **permutation** of 3 out of 5 objects.

**Step 2:** Divide by the permutation of the **number of objects** to be chosen from the total (3). This step eliminates the duplicates in finding the permutations.

**Step 3:** Cancel common factors and simplify.

$$\frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1} = 10$$

**Note:** Using the formula, find all your variables. The formula is  ${}_n C_r = \frac{n!}{(n-r)! r!}$ .  $n$  (the number you choose from) = 5,  $r$  (the number of objects arranged) = 3. Now, plug into the formula.

$${}_5 C_3 = \frac{5!}{(5-3)! 3!} = \frac{5!}{2! 3!} = \frac{5 \times 4 \times \cancel{3} \times \cancel{2} \times 1}{(2 \times 1) (\cancel{3} \times \cancel{2} \times 1)} = \frac{20}{2} = 10.$$

There can be 10 combinations of three letters from the set {a, b, c, d, e}.

**Find the number of combinations for each problem below.**

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|---|--|
| <ol style="list-style-type: none"> <li>1. How many combinations of 4 numbers can be made from the set of numbers {2, 4, 6, 7, 8, 9}?</li> <li>2. Johnston Middle School wants to choose 3 students at random from the 7th grade to take an opinion poll. There are 124 seventh graders in the school. How many different groups of 3 students could be chosen? (Use a calculator for this one.)</li> <li>3. How many combinations of 3 students can be made from a class of 20?</li> <li>4. Fashion Ware catalog has a sweater that comes in 8 colors. How many combinations of 2 different colors does a shopper have to choose from?</li> </ol> | <ol style="list-style-type: none"> <li>5. Angelo's Pizza offers 10 different pizza toppings. How many different combinations can be made of pizzas with four toppings?</li> <li>6. How many different combinations of 5 flavors of jelly beans can you make from a store that sells 25 different flavors of jelly beans?</li> <li>7. The track team is running the relay race in a competition this Saturday. There are 14 members of the track team. The relay race requires 4 runners. How many combinations of 4 runners can be formed from the track team?</li> <li>8. Kerri got to pick 2 prizes from a grab bag containing 12 prizes. How many combinations of 2 prizes are possible?</li> </ol> |
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## MORE COMBINATIONS

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Another kind of combination involves selection from several categories.

**EXAMPLE:** At Joe's Deli, you can choose from 4 kinds of bread, 5 meats, and 3 cheeses when you order a sandwich. How many different sandwiches can be made with Joe's choices for breads, meats, and cheeses if you choose 1 kind of bread, 1 meat, and 1 cheese for each sandwich?

### JOE'S SANDWICHES

<u>Breads</u>	<u>Meats</u>	<u>Cheeses</u>
White	Roast Beef	Swiss
Pumpernickel	Corned Beef	American
Light rye	Pastrami	Mozzarella
Whole wheat	Roast Chicken	
	Roast Turkey	

**Solution:** Multiply the number of choices in each category. There are 4 breads, 5 meats, and 3 cheeses, so  $4 \times 5 \times 3 = 60$ . There are 60 combinations of sandwiches.

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Find the number of combinations that can be made in each of the problems below.

- Angie has 4 pairs of shorts, 6 shirts, and 2 pairs of tennis shoes. How many different outfit combinations can be made with Angie's clothes?  
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- Raymond has 7 baseball caps, 2 jackets, 10 pairs of jeans, and 2 pairs of sneakers. How many combinations of the 4 items can he make?  
\_\_\_\_\_
- Claire has 6 kinds of lipstick, 4 eye shadows, 2 kinds of lip liner, and 2 mascaras. How many combinations can she use to make up her face?  
\_\_\_\_\_
- Clarence's dad is ordering a new truck. He has a choice of 5 exterior colors, 3 interior colors, 2 kinds of seats, and 3 sound systems. How many combinations does he have to pick from?  
\_\_\_\_\_
- A fast food restaurant has 8 kinds of sandwiches, 3 kinds of french fries, and 5 kinds of soft drinks. How many combinations of meals could you order if you ordered a sandwich, fries, and a drink?  
\_\_\_\_\_
- In summer camp, Tyrone can choose from 4 outdoor activities, 3 indoor activities, and 3 water sports. He has to choose one of each. How many combinations of activities can he choose?  
\_\_\_\_\_
- Jackie won a contest at school and gets to choose one pencil and one pen from the school store and an ice cream from the lunch room. There are 5 colors of pencils, 3 colors of pens, and 4 kinds of ice-cream. How many combinations of prize packages can she choose?  
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## CHAPTER 19 REVIEW

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Answer the following permutation and combination problems.

1. Daniel has 7 trophies he won playing soccer. How many different ways can he arrange them in a row on his bookshelf?
2. Missy has 12 colors of nail polish. She wears 1 color each day, 7 different colors a week. How many combinations of 7 colors can she make before she has to repeat the same 7 colors in a week?
3. Eileen has a collection of 12 antique hats. She plans to donate 5 of the hats to a museum. How many combinations of hats are possible for her donation?
4. Julia has 5 porcelain dolls. How many ways can she arrange 3 of the dolls on a display shelf?
5. Ms. Randal has 10 students. Every day she randomly draws the names of 2 students out of a bag to turn in their homework for a test grade. How many combinations of 2 students can she draw?
6. In the lunch line, students can choose 1 out of 3 meats, 1 out of 4 vegetables, 1 out of 3 desserts, and 1 out of 5 drinks. How many lunch combinations are there?
7. Andrea has 7 teddy bears in a row on a shelf in her room. How many ways can she arrange the bears in a row on her shelf?
8. Adrianna has 4 hats, 8 shirts, and 9 pairs of pants. Choosing one of each, how many different clothes combinations can she make?
9. The buffet line offers 5 kinds of meat, 3 different salads, a choice of 4 desserts, and 5 different drinks. If you chose one food from each category, from how many combinations would you have to choose?
10. How many pairs of students can Mrs. Smith choose to go to the library if she has 20 students in her class?